FLUID MECHANICS

Liquids and gases are together classified as fluids. They are the substances, which can flow. They cannot withstand shearing or tangential stress. They begin to flow when subjected to such stress.

PRESSURE:

A fluid at rest exerts an outward force on the inner surface of the container holding it. The force exerted per unit area on the surface of the container by the fluid is called pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$

$$P = \lim_{\Delta A \to 0}\left[\frac{\Delta F}{\Delta A}\right]$$

Pressure also exists in the interior of the fluid. Fluid within any region exerts an outward force on the fluid around it and fluid outside a region exerts an inward force on the fluid it surrounds. Fluid pressure can therefore be defined as the force exerted per unit area by a fluid on any surface in contact with it.

Pressure is measured in Nm^{-2} or Pascal or mm Hg or bar or torr. or atmosphere.

$1\ Nm^{-2} = 1$ Pascal

$1\ \text{bar} = 10^5\ \text{Pa}$

1 torr = 1 mm Hg

$1\ \text{atm} = 1.01325 \times 10^5\ \text{Pa} = 76$ cm of Hg = 760 torr

Pressure is a scalar quantity.

Density:

The mass per unit volume of a fluid is called its density

$$\rho = \frac{M}{V}$$

$$\rho = \lim_{\Delta V \to 0}\left[\frac{\Delta M}{\Delta V}\right]$$

Relative density or specific gravity is defined as

$$S = \frac{\text{density of substance}}{\text{Density of water at } 4°c}$$

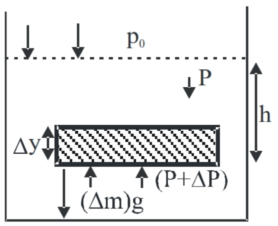
Density varies with temperature $\rho = \rho_0(1-\gamma\Delta T)$

Variation of density with change in pressure

$$\rho = \rho_0\left(1+\frac{\Delta p}{B}\right) = \rho_0(1+C\Delta p)$$

PRESSURE DUE TO A LIQUID COLUMN

Consider a liquid at rest in a container. Let us select a cylindrical portion of the liquid of cross sectional area A and height Δy. Since the liquid cylinder is in equilibrium,



$$(P+\Delta P)A - PA = \Delta mg$$

$$= \rho A g \Delta y$$

$$\boxed{\frac{\Delta P}{\Delta y} = \frac{dP}{dy} = \rho g}$$

If the density of the liquid is uniform, the pressure at depth h.

$$\int_{p_0}^{P} dP = \int_0^h \rho g dy$$

$$P = p_0 + \rho g h$$

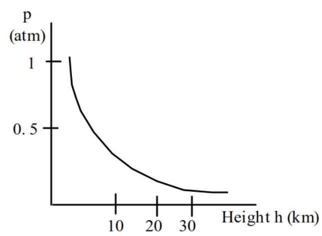
- The atmospheric pressure varies with height from surface of earth as

  $P = P_0 e^{-ha}$

  where $a = \frac{\rho_0 g}{P_0} = 8.55\,km^{-1} = 8.55$ Km.

  Atmospheric pressure decreases with height exponentially.

The constant a gives the difference in altitude over which the pressure drops by a factor of e. The pressure drops by a factor of 10 when the height increases by 20 km. Thus at the heights h = 20 km, 40 km above the sea level, the pressure would be 0.1 atm, 0.01 atm and so on.



- The pressure at a depth h in a liquid

  $p = p_0 + \rho g h$ is called hydrostatic pressure. The difference between hydrostatic and atmospheric pressure is called guage pressure.

  $P_g = p - p_0 = \rho g h$

- At a point inside the fluid, pressure acts in all directions and has the same value.

- If the liquid container is accelerated in the horizontal direction $\frac{dP}{dx} = -\rho a$

$$\frac{dp}{dx} = -\rho g$$

- The free surface of the liquid is inclined at an angle θ given by

$$\tan\theta = \frac{a}{g}$$

Force exerted by a fluid on the base of the container due to guage pressure

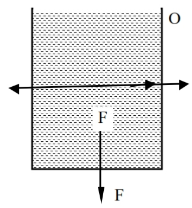
$F = \rho g V$ = Weight of liquid in container

- Force exerted by liquid on the vertical walls of the container due to guage pressure.

$$F = \rho g b \int_0^H h\,dh$$

where b = width of container ; H = height of liquid in container.

$$F = \frac{1}{2}\rho g b H^2$$



- The moment of this force about O

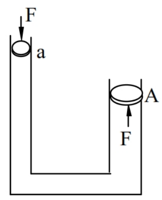
free surface of liquid,

$$M = \int_0^H h\,dF = F \times \frac{2H}{3}$$

Therefore total force acts at a point which is at a depth $\frac{2H}{3}$ from the free surface of liquid.

PASCAL LAW

Any change in pressure at one point of a confined fluid at rest is transmitted undiminished to every portion of the fluid and the walls of the container. An important application of this law is hydraulic lift. A small force f is exerted on the piston with small cross sectional area a. The resulting increase in pressure is $\Delta p = \frac{f}{a}$.



This change in pressure is transmitted through the liquid to a larger piston of area A. The force on the larger piston therefore.

$$F = \frac{f}{a}A. = \frac{A}{a}f$$

For a small force exerted on smaller piston a large force acts on larger piston. Hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the cross sectional areas of the two pistons.

Buoyancy : Archimedes Principle

When a body is wholly or partly immersed in a fluid. It experiences an upward thrust/force equal to the weight of the fluid displaced by it. This force is called force of buoyancy or buoyant force.

$$B = M_{\text{displaced fluid}} \times g = \text{Weight of fluid displaced}$$

This principle is Archimedes principle.

- There is a loss in weight of a body inside fluid due to buoyant force on body. Apparent weight of body in a fluid

  W' = True weight of body – Buoyant force

  $W' = W - B$

  $= V_b \rho_b g - V_l \rho_l g$

  V_l = volume of liquid displaced

  If the body is fully submerged

  $$W' = V(\rho_0 - \rho_l)g$$

- When a body floats in a liquid, the weight of the body is equal to the weight of the liquid displaced ($W' = 0$).
- The centre of gravity of the displaced liquid (called centre of buoyancy) lies vertically above or blow the centre of gravity of the body in equilibrium.

  If –

  (i) $\rho_b = \rho_l$ – body will float completely immersed in fluid any where in the fluid .

  (ii) $\rho_b > \rho_l$ – body will sink to the bottom of liquid

  (iii) $\rho_b < \rho_l$ – body will float partially immersed on the surface of fluid.

FLUID DYNAMICS

When the fluid velocity at any given point is constant in time the fluid motion is said to be steady. Flow of fluids is steady, usually at slow speeds.

If the element of fluids at each point has no net angular velocity about that point, the fluid flow is irrotational. Fluid flow is said to be unsteady or turbutant when the velocities of the fluid particles at any point changes erractically both in magnitude and direction with time.

During steady flow the trajectories of the fluid particles are in general curved paths know as streamlines. A streamline is a line drawn in the fluid such that the tangent to the streamline at any point is parallel to the fluid velocity at that point. The fluid velocity can vary from point to point along a streamline, but at any given point, the velocity remains constant in time. Steady flow is also called streamlined flow.

EQUATION OF CONTINUITY

If there are no sinks or sources of fluid in the tube, the mass of fluid flowing into the tube in a given time must be equal to mass of liquid flowing out of the tube in the same time. Therefore, mass flux is constant.

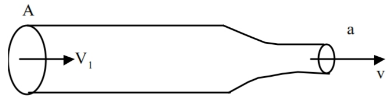
Mass flux $= \rho_1 Av = \rho_2 av =$ constant

$= \rho VA =$ constant

If the liquid is compressible $\rho_1 = \rho_2$

$AV = av$

$\boxed{AV = \text{constant}}$



This is equation of continuity

BERNOULLIS THEOREM

It is principle of conservation of energy applied to fluid flow. For streamline flow of an ideal fluid (nonviscous, incompressible) the sum of pressure energy, kinetic energy and gravitational potential energy per unit mass remains constant.

Pressure energy = PV

Pressure energy per unit mass $= \frac{P}{\rho}$

Kinetic energy per unit mass $= \frac{1}{2}V^2$

Gravitational potential energy per unit mass = gh

So $\frac{P}{\rho} + \frac{V^2}{2} + gh$ = constant

$\frac{1}{2}\rho V^2 + \rho gh =$ constant.

$\frac{P}{\rho g} + \frac{v^2}{2g} + h$ = constant.

APPLICATIONS OF BERNOULLI'S THEOREM

(i) Dynamic lift: Neglecting gravitational effects, velocity is higher at wings points where pressure is lower and vice versa of aeroplanes are so designed that velocity of wind on the upper part of the wings is higher compared to lower part. This results in a pressure deference with lower part having higher pressure. This pressure difference provides a dynamic lift.

(ii) Velocity of efflux : Consider a vessel containing a liquid upto some height H. A small hole is punched in the wall at a depth h below the free surface. The speed of liquid coming out of the hole is called the velocity of efflux of the liquid.

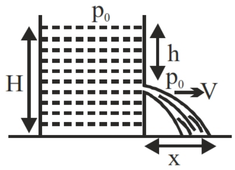
$$p_a + h\rho g + 0 = p_0 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh}$$

It equals the speed of an object fallen from rest through a height h.

Time taken by liqud to reach base levels,

$$t = \sqrt{\frac{2(H-h)}{g}}$$



The distance at which liquid hits the ground,

$$x = z\sqrt{h(H-h)}$$

VISCOCITY:

A fluid flows in layers. This is called laminar flow of fluid. Also, the liquid layer in contact with stationary surface of container remains at rest. As we move away from stationary surface the speed of liquid layers goes on increasing. There is therefore a relative motion between the layers of fluid.

When a layer of fluid slips or tends to slip on another layer in contact an internal resistance comes into play between layers which tends to destroy their relative motion. This internal resistance acting between layers of fluid is called viscous drag. This viscous drag acting between layers of fluid is

$$F = -\eta A \frac{dv}{dx}$$

where, A = Area of fluid layers;

$\frac{dv}{dx}$ = velocity gradient between liquid layers

η = constant of proportionality called coefficient of viscocity depending on the nature of fluid

negative sign indicates opposition to relative motion.

η is measured in Ns/m^2 or $dyne\text{-}s/cm^2$ or poise

FLOW OF FLUID THROUGH NARROW TUBE: POISEUILLE'S FORMULA

The rate of flow of a fluid of coefficient of viscocity η through a tube of length l and radius of cross section r when subjected to a perssure difference P is,

$$\frac{V}{t} = \frac{\pi P r^4}{8\eta l}$$

Stokes Law:

When a spherical body of radius r falls through a fluid of viscocity η with a velocity v a viscous drag acts on it given by,

$$F = 6\pi\eta r v$$

Terminal Velocity:

When a body falls freely through a fluid, its velocity goes on increasing and so does the viscous drag acting on it. When the viscous drag acting on the body equals, apparent weight of body, it begin to fall with a constant velocity called terminal velocity,

$$F = W - B$$

$$6\pi\eta r V_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$V_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Critical Velocity and Reynolds number:

At slow speeds, flow of fluid is streamlined. As velocity of fluid increases, the flow of fluid becomes turbulent. The largest velocity which allows a steady flow is called critical velocity.

The quantity,

$$N = \frac{\rho v D}{\eta}$$

is called the Reynolds number and determines the nature of flow of fluid.

If	$N < 2000$	–	Flow is steady
	$N > 3000$	–	Flow is turbulent
	$2000 < N < 3000$	–	Flow is unstable

This relation can be used to calculate critical velocity.